ISSN 2229-5518

Weak Forms of γ-Open Sets and New Separation Axioms

Hariwan Z. Ibrahim
Department of Mathematics, Faculty of Science, University of Zakho, Kurdistan-Region, Iraq
E-mail: hariwan_math@yahoo.com

Abstract— In this paper, we introduce some generalizations of γ-open sets and investigate some properties of the sets. Moreover, we use them to obtain new separation axioms.

Index Terms— γ-open, α-γ-open, pre-γ-open, β-γ-open, b-γ-open.

1 INTRODUCTION

Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless less otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by Cl(A) and Int(A), respectively.

Let (X, τ) be a space and A a subset of X. An operation γ on a topology τ [8] is a mapping from τ in to power set P(X) of X such that V ⊂ γ(V) for each V ∈ τ, where γ(V) denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [8] if for each x ∈ A, there exists an open set U such that x ∈ U and γ(U) ⊂ A. Then, τ_γ set denotes the of all γ-open set in X. Clearly $\tau_\gamma \subset \tau$. Complements of γ-open sets are called γ-closed. The τ_γ-interior [7] of A is denoted by $\tau_\gamma \subset \tau$. Complements of γ-open sets are τ_γ-Int(A) and defined to be the union of all γ-open sets of X contained in A. A topological X with an operation γ on τ is said to be γ-regular [8] if for each x ∈ X and for each open neighborhood V of x, there exists an open neighborhood U of x such that γ(U) contained in V. It is also to be noted that $\tau = \tau_\gamma$ if and only if X is a γ-regular space [8].

Definition 1.1

A subset A of a space X is said to be:

1. α-open [6] if A ⊆ Int(Cl(Int(A)));
2. semi-open [4] if A ⊆ Cl(Int(A));
3. pre-open [5] if A ⊆ Int(Cl(A));
4. β-open [1] if A ⊆ Cl(Int(Cl(A)));
5. b-open [2] if A ⊆ Int(Cl(A)) ∪ Cl(Int(A)).

In this paper we introduce and investigate the new notions called α-γ-open sets, pre-γ-open sets, β-γ-open sets and b-γ-open sets which are weaker than γ-open. Moreover, we use these notions to obtain new separation axioms.

Department of Mathematics, Faculty of Science, University of Zakho, Kurdistan-Region, Iraq.
E-mail: hariwan_math@yahoo.com

2 Weak Forms Of γ-Open Sets

Definition 2.1

A subset A of a space X is said to be:

1. α-γ-open if A ⊆ τ_γ-Int(Cl(τ_γ-Int(A)));
2. pre-γ-open if A ⊆ τ_γ-Int(Cl(A));
3. β-γ-open if A ⊆ Cl(τ_γ-Int(Cl(A)));
4. b-γ-open if A ⊆ τ_γ-Int(Cl(A)) ∪ Cl(τ_γ-Int(A)).

Lemma 2.2

Let (X, τ) be a topological space, then the following properties hold:
A subset A of a space X is said to be:

1. Every γ-open set is α-γ-open.
2. Every α-γ-open set is pre-γ-open.
3. Every pre-γ-open set is b-γ-open.
4. Every b-γ-open set is β-γ-open.

Proof

1. If A is a γ-open set, then A = τ_γ-Int(A). Since A ⊆ Cl(A), then A ⊆ Cl(τ_γ-Int(A)) and A ⊆ τ_γ-Int(Cl(τ_γ-Int(A))). Therefore A is α-γ-open.
2. If A is an α-γ-open set, then A ⊆ τ_γ-Int(Cl(τ_γ-Int(A))) ⊆ τ_γ-Int(Cl(A)). Therefore A is pre-γ-open.
3. If A is pre-γ-open, then A ⊆ τ_γ-Int(Cl(A)) ⊆ τ_γ-Int(Cl(A)) ∪ Cl(τ_γ-Int(A)). Therefore A is b-γ-open.
4. If A is b-γ-open, then A ⊆ τ_γ-Int(Cl(A)) ∪ Cl(τ_γ-Int(A)) ⊆ Cl(τ_γ-Int(Cl(A))) ∪ Cl(τ_γ-Int(A)) ⊆ Cl(τ_γ-Int(Cl(A))). Therefore A is β-γ-open.

Since every γ-open set is open, then we have the following diagram for properties of subsets.

γ-open→α-γ-open→pre-γ-open→b-γ-open→β-γ-open

↓ ↓ ↓ ↓ ↓

open → α-open → pre-open → b-open → β-open

The converses need not be true as shown by the following examples.

Example 2.3

Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$. Define an operation γ on τ by $\gamma(A) = A$ if $A = \{a, b\}$ and $\gamma(A) = X$ otherwise. Clearly, $\tau_\gamma = \{\varphi, \{a, b\}, X\}$. Then $\{a\}$ is an open set which is not β-γ-open.

Example 2.4

Let $X = R$ with the usual topology τ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = Q \cap [0, 1]$. Then A is a β-γ-open set which is not b-γ-open.

Example 2.5

Let $X = \{a, b, c\}$ and $\tau =\{\varphi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Define an operation γ on τ by $\gamma(A) = A$ if $A = \{b\}$ and $\gamma(A) = X$ if $A\neq \{b\}$. Clearly, $\tau_\gamma = \{\varphi, \{b\}, X\}$. Then $\{b, c\}$ is a b-γ-open set which is not pre-γ-open.

Example 2.6

Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, X\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then $\{a, c\}$ is a pre-γ-open set which is not α-γ-open.

Example 2.7

Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$. Define an operation γ on τ by $\gamma(A) = A$ if $A = \{a\}$ and $\gamma(A) = X$ if $A \neq \{a\}$ Clearly, $\tau_\gamma = \{\varphi, \{a\}, X\}$. Then $\{a, b\}$ is an α-γ-open set which is not γ-open.

Lemma 2.8

If U is an open set, then $Cl(U \cap A) = Cl(U \cap Cl(A))$ and hence $U \cap Cl(A) \subseteq Cl(U \cap A)$ for any subset A of a space X **[3]**.

Theorem 2.9

If A is a pre-γ-open subset of a space (X, τ) such that $U \subseteq A \subseteq Cl(U)$ for a subset U of X, then U is a pre-γ-open set.

Proof

Since $A \subseteq \tau_\gamma\text{-}Int(Cl(A))$, $U \subseteq \tau_\gamma\text{-}Int(Cl(A))$. Also $Cl(A) \subseteq Cl(U)$ implies that $\tau_\gamma\text{-}Int(Cl(A)) \subseteq \tau_\gamma\text{-}Int(Cl(U))$. Thus $U \subseteq \tau_\gamma\text{-}Int(Cl(A)) \subseteq \tau_\gamma\text{-}Int(Cl(U))$ and hence U is a pre-γ-open set.

Theorem 2.10

A subset A of a space (X, τ) is semi-open if A is β-γ-open and $\tau_\gamma\text{-}Int(Cl(A)) \subseteq Cl(Int(A))$.

Proof

Let A be β-γ-open and $\tau_\gamma\text{-}Int(Cl(A)) \subseteq Cl(Int(A))$. Then $A \subseteq Cl(\tau_\gamma\text{-}Int(Cl(A))) \subseteq Cl(Cl(Int(A))) = Cl(Int(A))$. And hence A semi-open.

Proposition 2.11

The intersection of a pre-γ-open set and an open set is pre-open.

Proof

Let A be a pre-γ-open set and U be an open set in X. Then $A \subseteq \tau_\gamma\text{-}Int(Cl(A))$ and $Int(U) = U$, by Lemma 2.8, we have $U \cap A \subseteq Int(U) \cap \tau_\gamma\text{-}Int(Cl(A)) \subseteq Int(U) \cap Int(Cl(A)) = Int(U \cap Cl(A)) \subseteq Int(Cl(U \cap A))$. Therefore, $A \cap U$ is pre-open.

Proposition 2.12

The intersection of a β-γ-open set and an open set is β-open.

Proof

Let U be an open set and A a β-γ-open set. Since every γ-open set is open, by Lemma 2.8, we have

$$
\begin{aligned}
U \cap A &\subseteq U \cap Cl(\tau_\gamma\text{-}Int(Cl(A))) \\
&\subseteq U \cap Cl(Int(Cl(A))) \\
&\subseteq Cl(U \cap Int(Cl(A))) \\
&= Cl(Int(U) \cap Int(Cl(A))) \\
&= Cl(Int(U \cap Cl(A))) \\
&\subseteq Cl(Int(Cl(U \cap A))).
\end{aligned}
$$

This shows that $U \cap A$ is β-open.

Proposition 2.13

The intersection of a b-γ-open set and an open set is b-open.

Proof

Let A be b-γ-open and U be open, then $A \subseteq \tau_\gamma\text{-}Int(Cl(A)) \cup Cl(\tau_\gamma\text{-}Int(A))$ and $U = Int(U)$. Then we have

$$
\begin{aligned}
&U \cap A \subseteq U \cap [\tau_\gamma\text{-}Int(Cl(A)) \cup Cl(\tau_\gamma\text{-}Int(A))] \\
&= [U \cap \tau_\gamma\text{-}Int(Cl(A))] \cup [U \cap Cl(\tau_\gamma\text{-}Int(A))] \\
&= [Int(U) \cap \tau_\gamma\text{-}Int(Cl(A))] \cup [U \cap Cl(\tau_\gamma\text{-}Int(A))] \\
&\subseteq [Int(U) \cap Int(Cl(A))] \cup [U \cap Cl(Int(A))] \\
&\subseteq [Int(U \cap Cl(A))] \cup [Cl(U \cap Int(A))] \\
&\subseteq [Int(Cl(U \cap A))] \cup [Cl(Int(U \cap A))].
\end{aligned}
$$

This shows that $U \cap A$ is b-open.

We note that the intersection of two pre-γ-open (resp. b-γ-open, β-γ-open) sets need not be pre-open (resp. b-γ-open, β-γ-open) as can be seen from the following example:

Example 2.14

Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a, b\}, X\}$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{a, c\}$ and $B = \{b, c\}$, then A and B are pre-γ-open (resp. b-γ-open, β-γ-open), but $A \cap B = \{c\}$ which is not pre-open (resp. b-γ-open, β-γ-open).

Proposition 2.15

The intersection of an α-γ-open set and an open set is α-open.

Theorem 2.16

If $\{A_k: k \in \Delta\}$ is a collection of b-γ-open (resp. α-γ-open, pre-γ-open, β-γ-open) sets of a space (X, τ), then $\cup_{k\in\Delta}A_k$ is b-γ-open (resp. α-γ-open, pre-γ-open, β-γ-open).

Proof

We prove only the first case since the other cases are similarly shown. Since $A_k \subseteq \tau_\gamma\text{-}Int(Cl(A_k)) \cup Cl(\tau_\gamma\text{-}Int(A_k))$ for every $k \in \Delta$, we have

$$
\begin{aligned}
&\cup_{k\in\Delta}A_k \subseteq \cup_{k\in\Delta}[\tau_\gamma\text{-}Int(Cl(A_k)) \cup Cl(\tau_\gamma\text{-}Int(A_k))] \\
&\subseteq [\cup_{k\in\Delta}\tau_\gamma\text{-}Int(Cl(A_k))] \cup [\cup_{k\in\Delta}Cl(\tau_\gamma\text{-}Int(A_k))] \\
&\subseteq [\tau_\gamma\text{-}Int(\cup_{k\in\Delta}Cl(A_k))] \cup [Cl(\cup_{k\in\Delta}\tau_\gamma\text{-}Int(A_k))] \\
&\subseteq [\tau_\gamma\text{-}Int(Cl(\cup_{k\in\Delta}A_k))] \cup [Cl(\tau_\gamma\text{-}Int(\cup_{k\in\Delta}A_k))].
\end{aligned}
$$

Therefore, $\cup_{k\in\Delta}A_k$ is b-γ-open.

We note that the intersection of two pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) sets need not be pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) as can be seen from the following example:

Example 2.17

Let $X = \{a, b, c\}$ and $\tau = P(X)$. Define an operation γ on τ by $\gamma(A) = A$ if $A = \{a, b\}$ or $\{a, c\}$ or $\{b, c\}$ and $\gamma(A) = X$ otherwise. Clearly, $\tau_\gamma = \{\varphi, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Let $A = \{a, b\}$ and $B = \{a, c\}$, then A and B are pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open), but $A \cap B = \{a\}$ which is not pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open).

Proposition 2.18

Let A be a b-γ-open set such that $\tau_\gamma\text{-}Int(A) = \varphi$. Then A is pre-γ-open.

A space (X, τ) is called a door space if every subset of X is open or closed.

Proposition 2.19

If (X, τ) is a door space and γ-regular, then every pre-γ-open set is γ-open.

Proof

Let A be a pre-γ-open set. If A is open, then A is γ-open. Otherwise, A is closed and hence A ⊆ τ_γ-Int(Cl(A)) = τ_γ-Int(A) ⊆ A. Therefore, A = τ_γ-Int(A) and thus A is a γ-open set.

3 New Separation Axioms

Definition 3.1

A topological space (X, τ) with an operation γ on τ is said to be:

1. pre-γ-T_0 (resp α-γ-T_0, b-γ-T_0, β-γ-T_0) if for each pair of distinct points x, y in X, there exists a pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) set U such that either x ∈ U and y ∉ U or x ∉ U and y ∈ U.
2. pre-γ-T_1 (resp. α-γ-T_1, b-γ-T_1, β-γ-T_1) if for each pair of distinct points x, y in X, there exist two pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) sets U and V such that x ∈ U but y ∉ U and y ∈ V but x ∉ V.
3. pre-γ-T_2 (resp. α-γ-T_2, b-γ-T_2, β-γ-T_2) if for each distinct points x, y in X, there exist two disjoint pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) sets U and V containing x and y respectively.

Remark 3.2

For a topological space (X, τ) with an operation γ on τ, the following properties hold:

1. If (X, τ) is α-γ-T_i, then it is pre-γ-T_i, for i = 0, 1, 2.
2. If (X, τ) is pre-γ-T_i, then it is b-γ-T_i, for i = 0, 1, 2.
3. If (X, τ) is b-γ-T_i, then it is β-γ-T_i, for i = 0, 1, 2.

Definition 3.3

A subset A of a topological space X is called a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set) if there are two pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) sets U and V such that U ≠ X and A = U \ V. It is true that every pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) set U different from X is a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set) if A = U and V = φ. So, we can observe the following.

Remark 3.4

Every proper pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) set is a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set).

Remark 3.5

For a topological space (X, τ) with an operation γ on τ, the following properties hold:

1. Every α-γD-set is pre-γD-set.
2. Every pre-γD-set is b-γD-set.
3. Every b-γD-set is β-γD-set.

Definition 3.6

A topological space (X, τ) with an operation γ on τ is said to be:

1. pre-γ-D_0 (resp. α-γ-D_0, b-γ-D_0, β-γ-D_0) if for any pair of distinct points x and y of X there exists a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set) of X containing x but not y or a γ-bD-set of X containing y but not x.
2. pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1) if for any pair of distinct points x and y of X there exist two pre-γD-sets (resp. α-γD-sets, b-γD-sets, β-γD-sets) U and V such that x ∈ U but y ∉ U and y ∈ V but x ∉V.
3. pre-γ-D_2 (resp. α-γ-D_2, b-γ-D_2, β-γ-D_2) if for any pair of distinct points x and y of X there exist disjoint pre-γD-sets (resp. α-γD-sets, b-γD-sets, β-γD-sets) G and E of X containing x and y, respectively.

Remark 3.7

For a topological space (X, τ) with an operation γ on τ, the following properties hold:

1. If (X, τ) is α-γ-D_i, then it is pre-γ-D_i, for i = 0, 1, 2.
2. If (X, τ) is pre-γ-D_i, then it is b-γ-D_i, for i = 0, 1, 2.
3. If (X, τ) is b-γ-D_i, then it is β-γ-D_i, for i = 0, 1, 2.

Remark 3.8

For a topological space (X, τ) with an operation γ on τ, the following properties hold:

1. If (X, τ) is pre-γ-T_i (resp. α-γ-T_i, b-γ-T_i, β-γ-T_i), then it is pre-γ-T_{i-1} (resp. α-γ-T_{i-1}, b-γ-T_{i-1}, β-γ-T_{i-1}), for i = 1, 2.
2. If (X, τ) is pre-γ-T_i (resp. α-γ-T_i, b-γ-T_i, β-γ-T_i), then it is pre-γ-D_i (resp. α-γ-D_i, b-γ-D_i, β-γ-D_i), for i = 0, 1, 2.
3. If (X, τ) is pre-γ-D_i (resp. α-γ-D_i, b-γ-D_i, β-γ-D_i), then it is pre-γ-D_{i-1} (resp. α-γ-D_{i-1}, b-γ-D_{i-1},β-γ-D_{i-1}), for i = 1, 2.

Theorem 3.9

A space X is pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1) if and only if it is pre-γ-D_2 (resp. α-γ-D_2, b-γ-D_2, β-γ-D_2).

Proof

Necessity. Let x, y ∈ X, x≠ y. Then there exist pre-γD-sets (resp. α-γD-sets, b-γD-sets, β-γD-sets) G_1, G_2 in X such that x ∈ G_1, y ∉ $G_1$and y ∈ G_2, x ∉ G_2 . Let G_1= U_1\$U_2$ and G_2= U_3\$U_4$, where U_1, U_2, U_3 and U_4 are pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) sets in X. From x∉G_2, it follows that either x ∉ U_3 or x ∈ U_3 and x ∈ U_4. We discuss the two cases separately.

i. x ∉ U_3. By y ∉ G_1 we have two subcases:

(a) y ∉ U_1. From x ∈ U_1\U_2, it follows that x ∈ U_1\(U_2 ∪ U_3), and by y ∈ U_3\ U_4 we have y ∈ U_3\ (U_1∪ U_4). Therefore (U_1\ (U_2∪ U_3)) ∩ (U_3\ (U_1 ∪ U_4)) = φ.

(b) y ∈ U_1 and y ∈ U_2. We have x ∈ U_1\U_2, and y ∈ U_2. Therefore (U_1\ U_2) ∩ U_2= φ.

ii. x ∈ U_3 and x ∈ U_4. We have y ∈ U_3\$U_4$ and x ∈ U_4. Hence (U_3\$U_4$) ∩ U_4= φ. Therefore X is pre-γ-D_2 (resp. α-γ-D_2, b-γ-D_2, β-γ-D_2).

Sufficiency. Follows from Remark 3.8 (3).

Theorem 3.10

A space is pre-γ-D_0 (resp. α-γ-D_0, b-γ-D_0, β-γ-D_0) if and only if it is pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0).

Proof

Suppose that X is pre-γ-D_0 (resp. α-γ-D_0, b-γ-D_0, β-γ-D_0). Then for each distinct pair x, y ∈ X, at least one of x, y, say x, belongs to a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set) G but y ∉ G. Let G = U_1\ U_2 where U_1 ≠ X and U_1, U_2 are two pre-γ-open (resp. α-γ-open, b-γ-open, β-γ-open) sets. Then x ∈ U_1, and for y ∉ G we have two cases: (a) y ∉ U_1, (b) y ∈ U_1 and y ∈ U_2. In case (a), x ∈ U_1 but y ∉ U_1. In case (b), y ∈ U_2 but x ∉ U_2. Thus in both the cases, we obtain that X is pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0).

Conversely, if X is pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0), by Remark 3.8 (2), X is pre-γ-D_0 (resp. α-γ-D_0, b-γ-D_0, β-γ-D_0).

Corollary 3.11

If (X, τ) is pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1), then it is pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0).

Proof

Follows from Remark 3.8 (3) and Theorem 3.10.

Definition 3.12

A point x ∈ X which has only X as the pre-γ-neighborhood (resp. α-γ-neighborhood, b-γ-neighborhood, β-γ neighborhood) is called a pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point).

Theorem 3.13

For a pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0) topological space (X, τ) the following are equivalent:

ISSN 2229-5518

1. (X, τ) is pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1).
2. (X, τ) has no pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point).

Proof

1 ⇒ 2. Since (X, τ) is pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1), then each point x of X is contained in a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set) A = U \ V and thus in U. By definition U≠ X. This implies that x is not a pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point).

2 ⇒ 1. If X is pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0), then for each distinct pair of points x, y ∈ X, at least one of them, x (say) has a pre-γ-neighborhood (resp. α-γ-neighborhood, b-γ-neighborhood, β-γ-neighborhood) U containing x and not y. Thus which is different from X is a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set). If X has no pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point), then y is not a pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point). This means that there exists a pre-γ-neighborhood (resp. α-γ-neighborhood, b-γ-neighborhood, β-γ-neighborhood) V of y such that V≠ X. Thus y ∈ V \ U but not x and V \ U is a pre-γD-set (resp. α-γD-set, b-γD-set, β-γD-set). Hence X is pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1).

Corollary 3.14

A pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0) space X is not pre-γ-D_1 (resp. α-γ-D_1, b-γ-D_1, β-γ-D_1) if and only if there is a unique pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point) in X.

Proof

We only prove the uniqueness of the pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point). If x and y are two pre-γ-neat points (resp. α-γ-neat points, b-γ-neat points, β-γ-neat points) in X, then since X is pre-γ-T_0 (resp. α-γ-T_0, b-γ-T_0, β-γ-T_0), at least one of x and y, say x, has a pre-γ-neighborhood (resp. α-γ-neighborhood, b-γ-neighborhood, β-γ-neighborhood) U containing x but not y. Hence U≠ X. Therefore x is not a pre-γ-neat point (resp. α-γ-neat point, b-γ-neat point, β-γ-neat point) which is a contradiction.

REFERENCES

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assuit Univ. 12: 77-90 (1983).

[2] D. Andrijevic, On b-open sets, Mat. Vesnik 48: 59-64 (1996).

[3] R. Engelking, General Topology, Heldermann Veriag Berlin, 2nd edition, (1989).

[4] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70: 36-41 (1963).

[5] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous functions, Proc. Math. Phys. Soc. Egypt 51:47-53 (1982).

[6] O. Njstad, On some classes of nearly open sets, Pacific J. Math. 15: 961-970 (1965).

[7] G. Sai Sundara Krishnan, A new class of semi open sets in a topological space, Proc. Ncmcm, Allied Publishers, New Delhi, 305-311 (2003).

[8] H. Ogata, Operation on topological spaces and associated topology, Math. Japonica, 36: 175-184 (1991).